A SPICE Model for Multiple Coupled Microstrips and Other Transmission Lines

VIJAI K. TRIPATHI, MEMBER, IEEE, AND JOHN B. RETTIG

Abstract —**A general multiple coupled line model compatible with standard CAD programs, such as SPICE, is presented. It is shown that the model can be used to help analyze and design coupled line (e.g., microstrip) circuits with linear, as well as nonlinear/time varying terminations, and to help study the pulse propagation characteristics in high-speed digital circuits.**

I. Introduction

A CONSIDERABLE amount of work has been done on the properties and applications of multiple coupled distributed parameter systems, including coupled transmission lines. The general solutions for the normal mode propagation constants, eigenvectors, impedances, and the network functions characterizing the $2n$-port for a coupled n-line structure are available in a matrix form [1]–[8]. In addition, for a smaller number of lines (e.g., two, three, or four lines) these properties can be expressed in an explicit closed form (e.g., [8]–[11]). Circuits can be analyzed and the design procedures can be formulated by utilizing the $2n$-port impedance, admittance, or general circuit parameters with the given terminations at the various ports of the structure. Dedicated computer programs for computing the scattering parameters and other properties of a terminated multiport are available for the analysis of such structures. Based on the solution of the coupled line equations, accurate models have been derived primarily for the case of coupled pairs of symmetric lines for microwave circuits and for multiple coupled lines for the analysis of the pulse propagation characteristics of the interconnections in digital circuits.

The model for the coupled pair of symmetrical lines consists of two uncoupled lines and coupling transformers at the input and the output ports and is now available in many microwave circuit design programs. For the case of general multiple coupled line structures (Fig. 1), the general coupled line analysis has led to the model consisting of uncoupled lines coupled at the input and the output ends by a congruent transformer bank [1]. This model has been modified by using the method of characteristics [2], [3], and has been applied to the case of the interconnections in high-speed digital circuits that are best represented as coupled transmission lines [12], [13]. All these models, together with the ones presented here, are mathematically identical in that they all represent the exact solution of the coupled transmission-line equations.

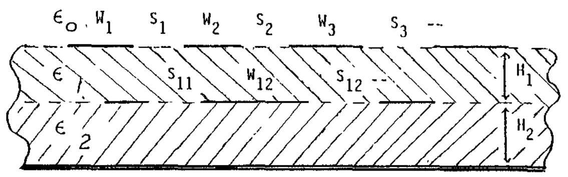

Fig. 1. Cross-sectional view of a generic multiple coupled line structure.

In this paper, a multiple coupled line model consisting of uncoupled lines and linear dependent current and voltage sources is presented. Since all of these elements are available in most CAD programs used in the design of digital as well as microwave circuits, the model can be easily incorporated in the form of subcircuits making the coupled line $2n$-port a standard circuit element from a computer-aided circuit analysis and design point of view.

II. The Model

The normal mode analysis of general multiple uniformly coupled transmission lines is reasonably well known (e.g., [1]–[8]) and is reviewed here in a simple, concise form. The voltages and currents on a lossless n-line system are described by the transmission-line equations

$$\frac{\partial v]}{\partial z} = -[L]\frac{\partial i]}{\partial t} \tag{1a}$$

$$\frac{\partial i]}{\partial z} = -[C]\frac{\partial v]}{\partial t} \tag{1b}$$

where vectors

$$v] = [v_1, v_2 \cdots v_n]^T$$
$$i] = [i_1, i_2 \cdots i_n]^T$$

represent voltages and currents on the lines, T represents the transpose, and $[L]$ and $[C]$ the inductance and capacitance matrices whose elements represent per unit length

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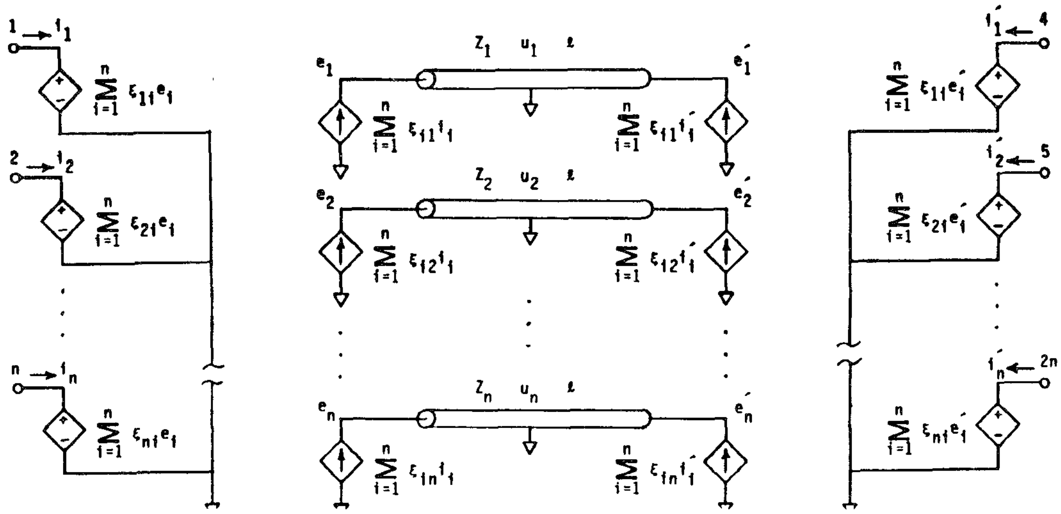

Fig. 2. The multiple coupled line circuit model.

self and mutual parameters of the lines. It should be noted that $[L]$ is a positive definite and $[C]$ is a hyperdominant matrix.

Since the uniform system under consideration is time invariant, we can consider the equations in the Fourier transform or frequency domain and define $V]$ and $I]$ as the corresponding voltage and current vectors. Then, for this $\exp\cdot[j(\omega t-\beta z)]$ variation, (1a) and (1b) are easily decoupled leading to the eigenvalue equations for voltages and currents as given by

$$[[L][C]-\lambda[U]v]=0 \qquad (2a)$$

$$[[C][L]-\lambda[U]]I]=0 \qquad (2b)$$

where $\lambda \triangleq \beta^2/\omega^2$ and $[U]$ is the unit matrix.

Now, since $[L]$ $[C]$ and $[C]$ $[L]$ are adjoint, the eigenmodes of voltages and currents share the same eigenvalues (i.e., the same phase velocities) and are B-orthogonal. The above equations can be expressed as

$$[C]V]=\lambda[L]^{-1}V] \qquad (3a)$$

and

$$[L]I]=\lambda[C]^{-1}I]. \qquad (3b)$$

The structures of interest consist of a single or multilayered dielectric medium whose magnetic properties are the same as free space. That is, if $[C_0]$ and $[L_0]$ are the capacitance and inductance matrices for the same structure with dielectric removed, then

$$[L]=[L_0]=\mu_0\epsilon_0[C_0]^{-1}. \qquad (4)$$

Equations (3a) and (3b) can be expressed in terms of capacitance matrices only by utilizing (4). They represent the generalized matrix eigenvalue and eigenvector problems of the type $[A]\ x]=\lambda[B]\ x]$ found in many books on linear algebra.

Let the voltage eigenvector matrix be defined as $[M_V]$; then, the current eigenvector matrix $[M_I]$ is given by [1]

$$[M_I]=\left[[M_V]^T\right]^{-1}. \qquad (5)$$

Substituting $v]=[M_V]\ e]$ and then $i]=[M_I]\ j]$ in the transmission line, (1a) and (1b) lead to

$$\frac{\partial e]}{\partial z}=-\operatorname{diag}[L_k]\frac{\partial}{\partial t}j] \qquad (6a)$$

$$\frac{\partial j]}{\partial z}=-\operatorname{diag}[C_k]\frac{\partial}{\partial t}e] \qquad (6b)$$

where $\operatorname{diag}[L_k]$ and $\operatorname{diag}[C_k]$ are diagonal matrices as given by

$$[L_k]=[M_V]^{-1}[L]\left[[M_V]^T\right]^{-1}=\frac{1}{u_k^2C_k} \qquad (7a)$$

$$[C_k]=[M_V]^T[C][M_V]. \qquad (7b)$$

Equations (6a) and (6b) represent n uncoupled transmission lines having equivalent capacitances and inductances as given above resulting in a mode characteristic impedance $Z_k=\sqrt{L_k/C_k}$. u_k is the phase velocity of the kth mode and is related to the kth eigenvalue λ_k. The above decoupled equations (6a) and (6b), together with the orthogonality between current and voltage eigenvectors as represented by [5], that is, the transformation according to

$$\begin{bmatrix} v] \\ j] \end{bmatrix}=\begin{bmatrix} [M_V] & 0 \\ 0 & [M_V]^T \end{bmatrix}\begin{bmatrix} e] \\ i] \end{bmatrix} \qquad (8)$$

lead to an equivalent circuit model shown in Fig. 2 for a general n line system. It should be noted that (8) represents a congruent transformation and has also been implemented by an ideal transformer bank [1]. The models consist of elements represented by the relationships given by (8) at the input and output ports that are coupled to each other via the uncoupled lines represented by (6).

The model shown in Fig. 2 is a circuit that defines (6) and (8), that is, the solution of coupled transmission-line equations (1a) and (1b). The model consists of uncoupled lines and linear-dependent sources and is compatible with most CAD programs including SPICE. Much of our work presented here was done on SPICE since all the model

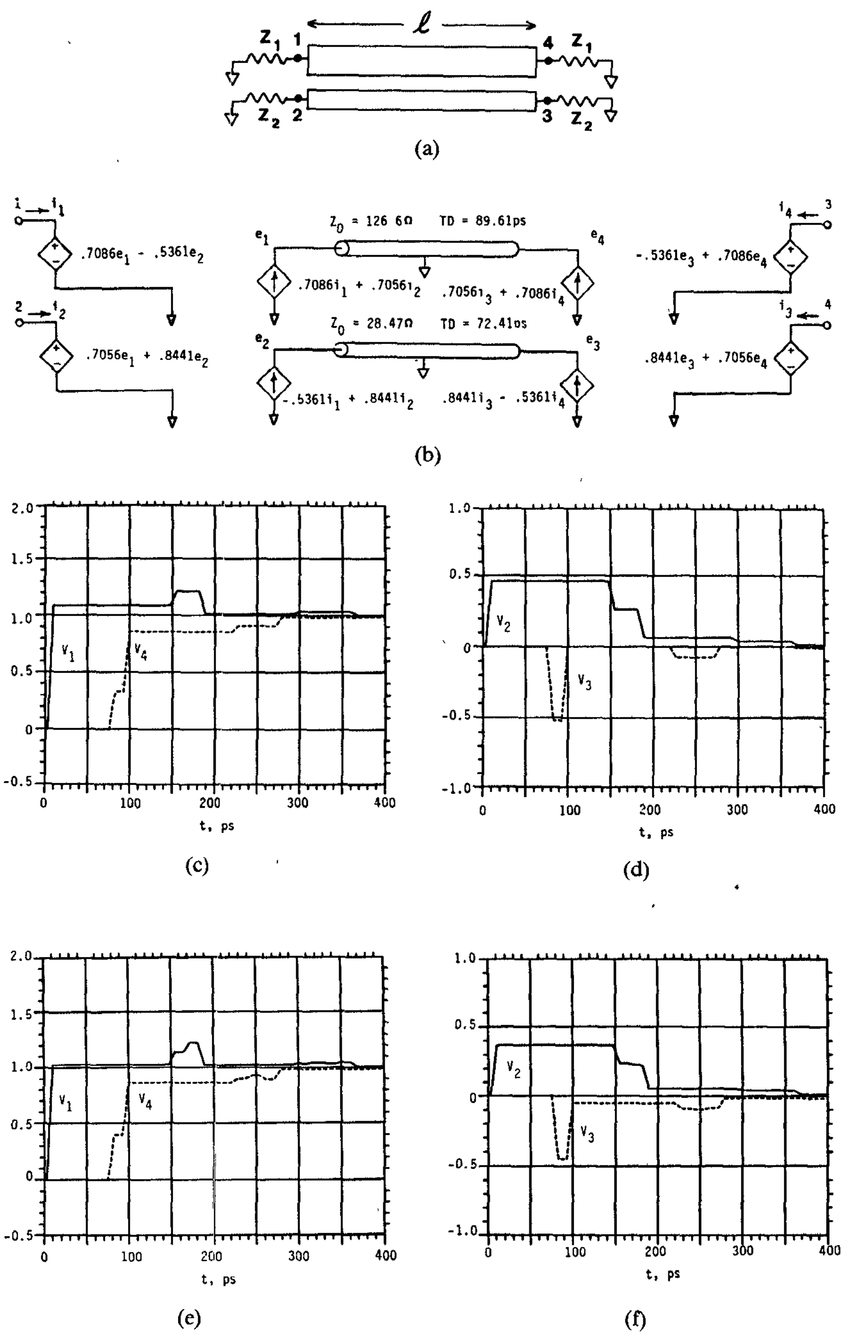

Fig. 3. Step response of the asymmetric coupled microstrip four-port. (a) Schematic of an asymmetric coupled line four-port. (b) The SPICE model for an asymmetric coupled microstrip four-port $W_1/H = 2W_2/H = 0.46$, $S/H = 0.038$, $\epsilon_r = 9.8$. (c) and (d) Characteristic nonmode converting termination $Z_1 = 46.8\ \Omega$, $Z_2 - 73.4\ \Omega$ [17]. (e) and (f) 50-Ω terminations.

elements are available in SPICE and that it can be used to analyze multiports in both the time and frequency domains. The input parameters used for the SPICE subcircuit (model) are as follows.

Length of the Lines.

Velocity of propagation of each normal mode of the system: $u_k = 1/\sqrt{\lambda_k}$, where λ_k is the eigenvalue of the $[[L][C]]$ matrix. $k = 1, 2, \cdots, n$.

The uncoupled normal mode characteristic impedances $(Z_k = 1/(u_k C_k))$ for each mode.

Voltage eigenvector matrix elements ξ_{ij}, $i, j = 1, 2, \cdots, n$.

The normal mode velocities, impedances, and eigenvector matrix elements required to construct the model are derived from the capacitance and inductance matrices of the coupled line system as shown above. Several techniques are available to compute these coefficients for multiple coupled line structures in layered as well as simple medium (e.g., [6], [15], [16]).

III. Results and Discussion

The above model can be used to analyze and formulate design procedures for symmetrical, nonsymmetrical, and multiple coupled line circuits such as four- and six-port

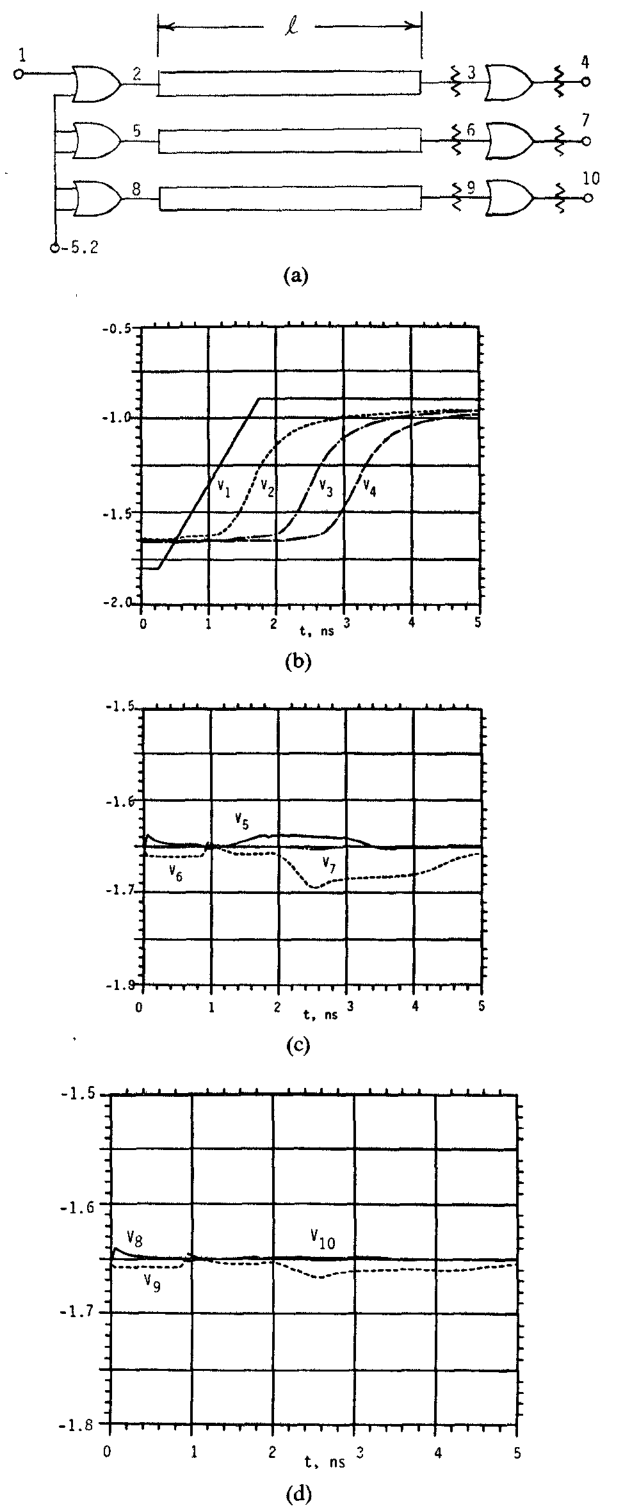

Fig. 4. Step response of a three-line structure on Alumina terminated in ECL-OR gates. $W/H = S/H = 1$. The termination symbols represent 50 Ω and consist of two resistors in parallel (81 Ω to ground, and 130 Ω to −5.2V)

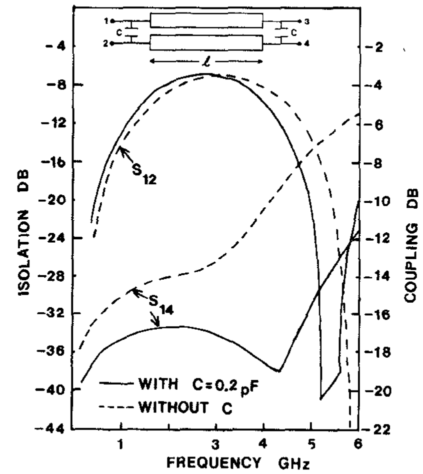

Fig. 5. Isolation and coupling for an edge-coupled 3-dB microstrip coupler on Alumina with and without a velocity equalizing lumped capacitor [18], [19].

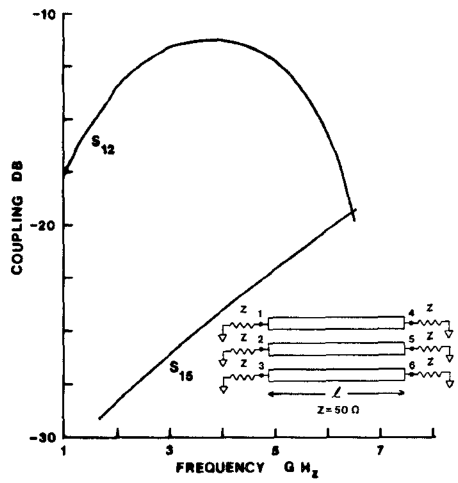

Fig. 6. The computed scattering parameters of a three-line six port 10-dB (nominal) coupler. $W/H = 0.7$, $S/H = 0.3$, $\epsilon_r = 9.7$, $1 = 1$ cm.

couplers and two-port filters and transformers. The analysis and design capabilities are only limited by the CAD program being used and not the model. It should be noted that for both the frequency-domain and the time-domain analysis of a given analog or digital system, the coupled line multiport can be included as an integral part of the system and can be incorporated in the overall computer-aided design of the system. This would be much more effective than the commonly used practice of first analyzing or designing the multiports based on idealized terminations, and then trying to assess the deviations in its performance when it is inserted in or coupled to a real system.

Several representative coupled microstrip structures have been analyzed for their frequency- and time-domain characteristics with linear as well as nonlinear terminations. The objectives range from an accurate representation of elements that are best represented as interconnected coupled microstrips, such as spiral inductors and interdigital capacitors, to the design of circuit elements consisting of symmetrical, nonsymmetrical, and multiple coupled lines such as filters and couplers. Some of these results are shown in Figs. 3 through 7.

Fig. 3 shows the SPICE model, model parameter values, and the step response of a general nonsymmetrical coupled lines four-port terminated in its characteristic nonmode converting values as given by $Z_1 = \sqrt{Z_{c1}Z_{\pi 1}}$ and $Z_2 = \sqrt{Z_{c2}Z_{\pi 2}}$ [17]. The step response of the same structure

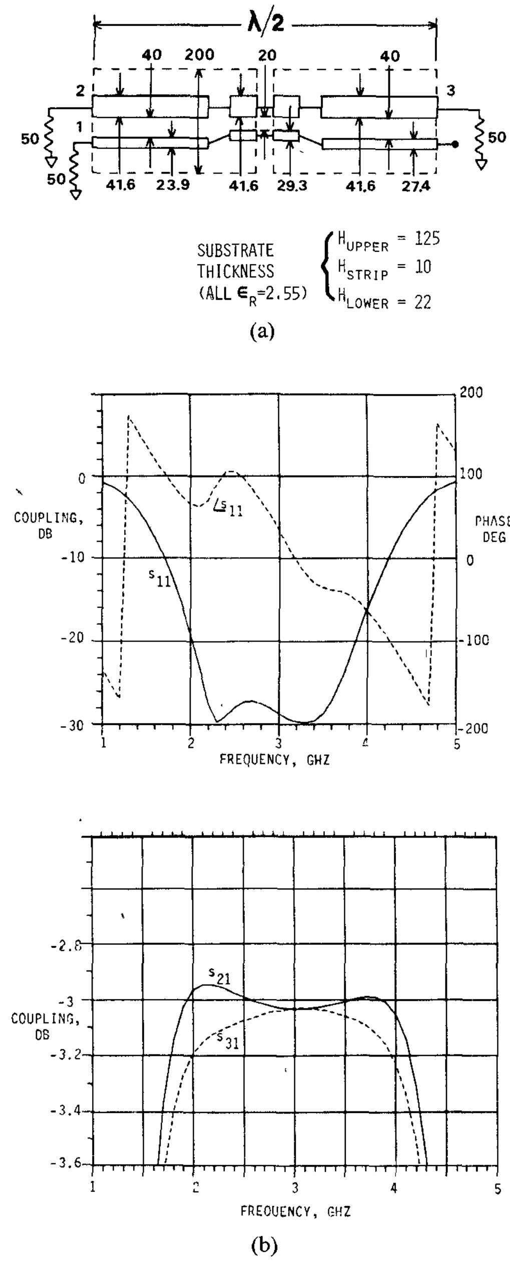

Fig. 7. (a) A microstrip balun (similar to the one in [20]). (b) Computed scattering parameters of the balun.

terminated in 50 Ω is also given for comparison. The effect of unequal normal mode phase velocities results in a finite pulse at the isolated port even for the properly terminated four-port (Fig. 3(c) and (d)). In addition, the results obtained with a 50-Ω termination (Fig. 3(e) and (f)) also demonstrate the effects of mismatch on the response of the four-port.

It is highly desirable to have the ability to use nonlinear circuit terminations with the coupled line model for the analysis and design of active circuits and for many other applications such as the computation of the pulse propagation characteristics including cross talk in planar or multilevel interconnections in high-speed digital circuits. In order to illustrate the application of the model with nonlinear terminations, a three-line structure terminated in logic gates as shown in Fig. 4 is analyzed. These gates are similar to the 100K ECL family and are constructed with the BJT transistor models available in SPICE 2G.5. The rise-time and gate propagation delays correspond to subnanosecond performance. The topology chosen uses terminations at the receiver end of the lines but no source terminations and single-ended transmission mode. The signal was applied to the gate on the outside lines and the output at all the other ports was monitored and is shown in Fig. 4. The effect of the interaction between the gates and the lines representing the interconnects is automatically included in the results which in addition to crosstalk also give us accurate propagation delays and other pulse propagation characteristics.

The frequency response of an edge-coupled 3-dB coupler is shown in Fig. 5 with and without a velocity compensating lumped element [18], [19] and the frequency response of a nominal 10-dB six-port coupler is shown in Fig. 6. Both of these are computed in a straightforward manner by utilizing the coupled line models. Fig. 7 shows the response of a microstrip balun which, as shown in the same figure, is somewhat similar to Laughlin's structure [20]. These sets of examples are presented to demonstrate the applications of the model presented here for the design of linear circuits in frequency domain with linear terminations.

It should be noted that dedicated computer programs for the analysis of lossy as well as lossless lines applicable to linear and nonlinear terminations based on the method of characteristics (e.g., [2], [3]) and many others based on the immitance parametes of the $2n$-port are available for the design of digital and microwave circuits. The primary objective of this work is to show that most CAD programs for active, microwave, and digital circuit design including SPICE, for which MESFET [21] and other device models have also been reported recently, can also incorporate multiple coupled strips as a subcircuit.

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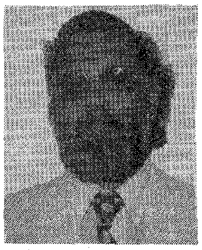

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